DISPLACEMENT INTERFEROMETRY IN CONNECTION WITH U-TUBES

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1. *Introduction.*—A variety of constants in physics may be found from the relative heights of two communicating columns of liquid. This is for instance the case in the classical experiment of Dulong and Petit on the thermal expansion of liquids. Again if one of the tubes is subject to a special force acting in the direction of its axis, this force in its bearing on the liquid may be evaluated from the resulting difference of heads of the columns. Thus one tube may be surrounded by a magnetizing helix and the effect of the axial magnetic field on the liquid in question (i.e., the susceptibility) found from the displacement of its surface by the presence and absence of the field; etc. It seemed to me worth while therefore to test whether it would be possible to measure small displacements of this kind by passing the two component beams of a displacement interferometer axially *through* the two columns respectively, and to measure the differential effects in question in terms of the resulting displacements of fringes.

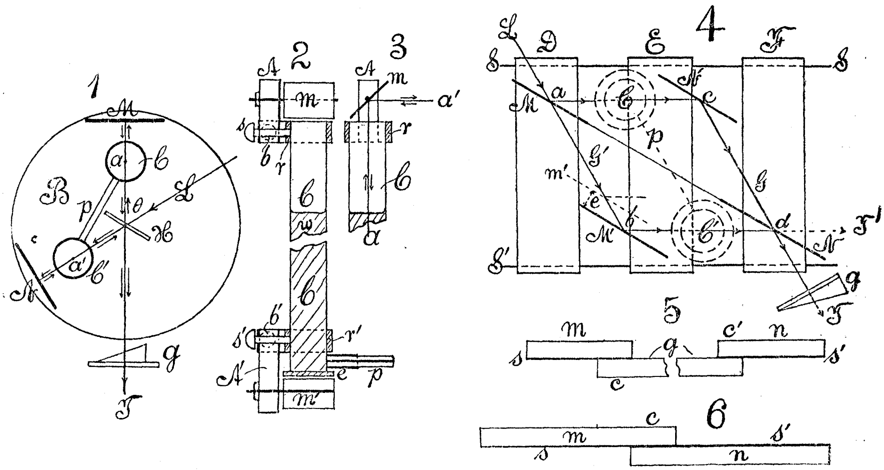
2. *Apparatus.*—The interferometer used was of the same form as that before described (these PROCEEDINGS, **3**, 1917, 117), B in figure 1 being a heavy iron block, one foot in diameter and 1.5 inches thick, on which the mirrors M, N (the latter and preferably both on micrometers) are securely mounted with the usual direct rough and elastic fine adjustment for horizontal and vertical axes. A beam of parallel white rays L arrives from a collimator (not shown) and impinges on the half silver plate H, to be reflected and transmitted at a convenient angle θ (about 60°), thus furnishing the two component beams which are to traverse the limbs of the U-tube.

The vertical columns of this tube are shown at C and C' (with accessory mirrors removed) and they are joined to the capillary tube p near the bottom of C and C'. Details will be given in connection with figures 2 and 3.

The ray HM strikes a mirror symmetrically at 45° to the vertical below C, is thence reflected upward along the axis a striking another mirror above also symmetrically at 45° and parallel to the former, whence it is reflected to the opaque mirror M. The latter reflects the ray normally back so that it retraces its path as far as H, by which plate it is now transmitted to be observed by the telescope at T. Sim-

ilarly the transmitted component ray HN is guided by suitable reflectors at 45°, so as to take the path $Ha'Na'HT$, thus passing axially (a') through the tube C'.

It is necessary that the U-tube CpC' be mounted independently of the block B on suitable bracket or arm attached to the pier. Otherwise any manipulation at N will disturb the surfaces of water in C and C'. Ordinary clamps admit of raising or lowering or rotating CC' satisfactorily, always providing that it shall not touch B. The telescope at T is also mounted apart from B on the table below. The direct vision prism grating g is placed immediately in front of the objective and swivelled (as described, loc. cit.) so that either the white slit images or their spectra may be seen in the field of view, according as g is rotated aside or is in place.



In figure 2 a front sectional elevation of one of the shanks of the U-tube is given with all appurtenances, and a similar sectional elevation at right angles to the former is added in figure 3 for the top of the tube. In figure 2 the mirrors m' and m are on horizontal axes and the component ray coming from behind the diagram strikes m' below, is reflected axially upward through CC, impinging on the mirror m (also on a horizontal axis) whence it is reflected horizontally toward the front of the diagram. The ray a and mirror m are given more clearly in figure 3. The lateral capillary tube appears at p and the tube C is closed below with a plate of glass e, cemented in place.

To mount the mirrors m, m', snugly fitting rings r and r' encircle the tube C near its top and bottom and can be fixed by the set screws s and s'. In virtue of these rings the mirrors m, m', may be rotated at pleasure around the vertical axis a of CC. The horizontal axis of the

mirrors m, m', rotates at pleasure in the vertical arms A, A' of square brass tube. A, A' in turn may be slightly swivelled about the horizontal axis b, b', in a rigid lateral projection of the rings r, r'. Thus m, m' are capable of rotation around 3 axes normal to each other, and adequately clamped in any position.

The component ray HN may be adjusted to the center of the lower mirror m' by placing the collimator L and then guided axially by $m'mN$ as described each being adjustable. The component ray HM may be similarly adjusted to the center of the lower mirror m' (at 45°) by (slightly) rotating the half silver plate, H (on horizontal and vertical axes) and then guided axially by $m'mM$. As a whole the adjustment is difficult, though it need not be much refined. Clear white slit images in the telescope T are an adequate criterion.

In the absence of a liquid in CC figure 2, the fringes are easily found after careful preliminary measurement and they are strong and satisfactory. When this adjustment is given the presence of liquid in CC, if the two columns are of nearly equal length, does not much modify the adjustment. In fact the fringes were found much more easily than I anticipated and in quiet surroundings they are strong and fine. It is necessary however that the tube CC should be of sufficient width to avoid all curvature due to capillarity, at least in the axis. Tubes 2 cm. in diameter and 10 cm. long of thin brass were first tried, but proved to be too narrow. No sharp slit images could be obtained with reasonable care as to setting the mirrors. Thereafter tubes 4 cm. in diameter were used and these proved to be satisfactory. Slit images were sharp and parallel and could be easily brought to coincide.

3. *Equations.*—Some estimate of the increments to be anticipated may be given here, and expressed in terms of the Dulong-Petit experiment. If α is the mean coefficient of expansion of water at the temperature in question and ΔH the increment of the head H corresponding to the temperature difference Δt between the columns

$$\Delta H = \alpha H \Delta t \tag{1}$$

Again if ΔN corresponding to ΔH is the displacement of center of ellipses at the wave length λ and μ the index of refraction of water, so that $\mu = A + B/\lambda^2$, nearly,

$$\Delta H = \frac{\Delta n}{\mu - 1 + 2B/\lambda^2} \tag{2}$$

Hence Δt may be computed as

$$\Delta t = \frac{\Delta n}{(\mu - 1 + 2B/\lambda^2)\alpha H} \tag{3}$$

Since the value of ΔN is within 10^{-4} cm. and $H = 10$ cm. in the above apparatus, we may further write at mean temperatures (25°) $\alpha = 2.5 \times 10^{-4}$; $\mu = 1.333$; $B = 10^{-11} \times 3.1$, $2B/\lambda^2 = 0.018$ at the D line. Thus $\mu - 1 + 2B/\lambda^2 = .351$ and $\Delta t = 10^{-4}/3.51 \times 2.5 \times 10^{-4} \times 10 = 0.114°$. In other words in case of tubes 10 cm. long, the effect of a difference of temperature of about 0.1° between the tubes should be easily observable by mere displacement, whereas a difference of less than 0.03° would be equivalent to the passage of one interference ring.

Again from equation (2), if $\Delta N = 10^{-4}$ cm. then $\Delta H = 10^{-4}/.351 = 10^{-4} \times 2.8$ cm. or about 9×10^{-5} cm. per vanishing interference ring are the displacements to be anticipated. These are equivalent to pressures of about 0.3 and 0.1 dynes per cm².

A number of experiments were made with this apparatus but for these there is no space here.

5. *Jamin's Interferometer*.—The ease with which the Michelson interferometer may be adjusted and its remarkable adaptability have led to its general preference over the older form of Jamin. Nevertheless the latter furnishes two parallel rays which for such purposes as the present are desirable. Hence if the four faces of the interferometer be separated in the manner suggested by Mach, a very available form of interferometer is obtained. But the trouble with the arrangement is the difficulty of adjusting the *four* surfaces. Not only are the centers of ellipses liable to be remote from the center of the field, but it is often hard, without special equipment, to even find the fringes.

If however the device which I suggested elsewhere is adopted, i.e., if figure 4, the half silvered plates, M, N, are at the ends of a simple strip of plate glass, so that rays terminating in M M' N N' after adjustment necessarily make a rhombuslike figure symmetrical to MN the fringes are found at once: for they appear when the white slit images in T coincide horizontally and vertically and the rays bd and cd intersect in the common point d. Hence the mirrors, M', N', should be on carriages D, F, adapted to move on the parallel slides S, S'. M, N may also be put on a carriage E though this is not necessary. S, S' need not be parallel to ac or bd. If the mirror M' and N' are wide, considerable latitude of adjustment is thus obtained.

If M N is half silvered on the same side (i.e., toward N') a compensator is needed in ac or cd, if path difference is to be annulled (symmetry). If however M is half silvered on the N' side and N on the M' side, no compensator is required. In the latter case however, if ordinary plate glass is taken, M and N are not quite parallel and the ellipses will be

eccentric. This however is not necessarily a disadvantage, unless the strip MN is excessively wedgeshaped.

The ellipses obtained are usually long vertically, so that the fringes soon become straight and the rotation is extremely rapid whenever the center of ellipses is out of the field. It is therefore possible to adjust relative to horizontal fringes (parallel to shadow of wire across slit), as these incline very obviously for a displacement of less than 10^{-4} cm. and rapidly become vertical. For this reason it makes little difference whether the half silvers are on the same or on opposite sides, or whether observation be made at T (*cd* prolonged) or at T' (*bd* prolonged). Moreover the plate MN may be conveniently constructed as in figure 5, of two mirrors m, n, attached to the clear strip of plate glass, g, by aid of strong steel clips at c, c'. With the half silvers s, s' on the same side, the wedge angle of the glass is excluded. For shorter diagonals, the plan of figure 6 with the silver surfaces s, s' held together by clips at c is preferable.

If the mirror M', figure 4, is displaced a distance e where a glass plate compensator of thickness E, and refraction constants μ and B is introduced normally either into *ab* or *bd*, the equation is easily seen to be, at wave length λ

$$E\ (\mu - 1) + 2\ B/\lambda^2 = 2e \cos \theta$$

where θ is the angle of reflection at M. Using the plate $E = 0.434$ cm. treated above, the first member is 0.2428 cm. Values of e of 0.2420, 0.2409, 0.2427 were roughly obtained. Hence the mean value of θ should be about 60°, as it actually was. Certain outstanding difficulties may be met by making MN, figure 4, the *short* diagonal of the rhombus and using the strip figure 6. In such a case θ at M' is small and in view of the nearly normal reflection at M and N relatively little reflection comes from naked glass, sliding is largely avoided and no compensator is necessary. In this case the fringes for no path difference are actually black horizontal lines on a colored ground and far enough apart that 1/10 fringe could easily be estimated. A test experiment with the above plate showed $e = 0.1244$ cm. corresponding to the small angle θ a little over 12°.

When the U-tube CC', figures 2 and 4, is introduced, the strip $M\ N$ will have to be at a considerable angle (about 45°) to the horizontal, so as to raise the N end about 15 cm. above the M end, corresponding to the height of m above m' in figure 2. The new condition however in no way changes the general procedure. In case of figure 5 the mirror N'

must be high and M' low. If C' is at G' the whole of each component beam may be caught and passed through the respective shanks of the U-tube. The fringes are strong, easily found and large, so that the center of ellipses is not far outside of the field of the telescope.

Finally if the connecting tube p is nearly horizontal when in place, the fringes are usually found at about the same position of the micrometer (at M') after the liquid is introduced into the U-tube.

Experiments were also made with this apparatus. Displacement interferometers in which the rays do not retrace their respective paths have an important special property which I wish to accentuate in conclusion. If either opaque mirror is displaced on its micrometer normal to itself or if a plate compensator rotates in one beam on a vertical axis, the center of ellipses moves parallel to the length of the spectrum. If however the plate compensator rotates on a *horizontal* axis, the center of ellipses moves nearly *transversely* to the length of the spectrum. The phenomenon is quite sensitive. To this result I shall return in a succeeding note, in connection with the development of the Jamin design for displacement interferometry.

The present note will be presented in more extended form in a report to the Carnegie Institution of Washington.

ATTEMPT TO SEPARATE THE ISOTOPIC FORMS OF LEAD BY FRACTIONAL CRYSTALLIZATION

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Although the complete inseparability of isotopes by chemical means has been frequently asserted, the evidence on which this assertion is based has always seemed insufficient. The methods used have been fractional crystallization and precipitation, but these processes have seldom been carried out more than ten times in a particular case, and frequently six or seven crystallizations have been thought a sufficiently thorough test of inseparability. A search of the literature revealed only one investigation, that of Radiothorium and Thorium by McCoy and Ross,[1] where as many as one hundred repetitions of a given process had been made.

It seemed worth while, therefore, to apply to the important generalization of Fajans, Russell, Fleck and Soddy a more searching test carrying the fractionation further, and using as a criterion of success not only the measurement of radioactivity but also the determination of atomic weight.